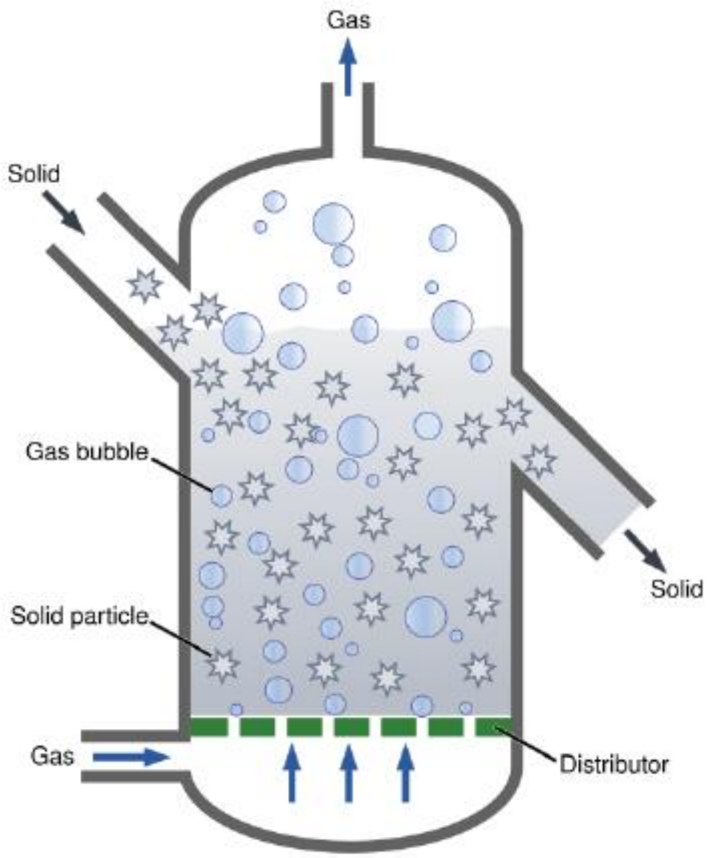
Fluidization

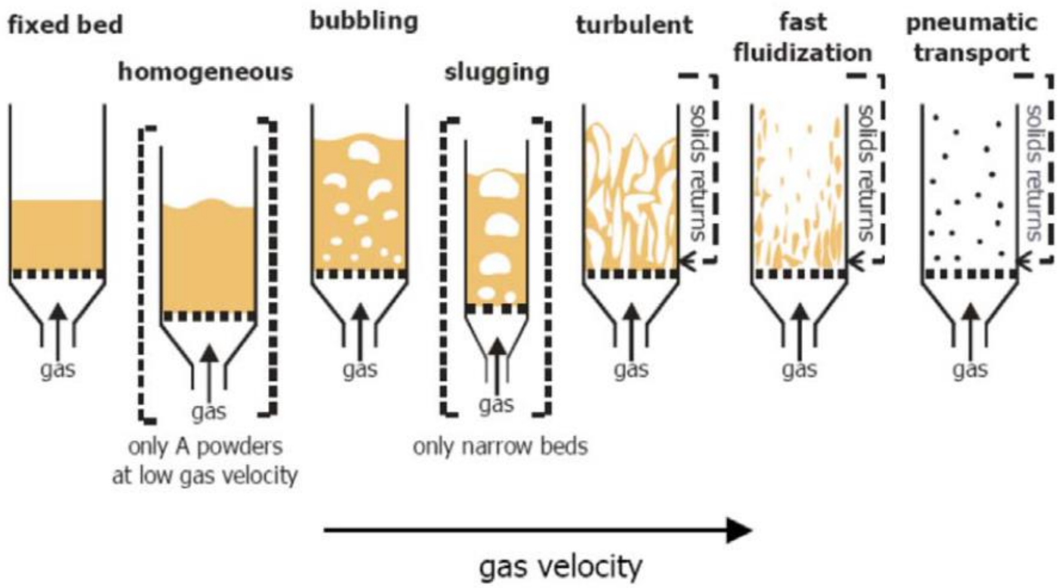
When a fluid is passed upwards through a bed, the pressure drop is the same as that for downward flow at relatively low rates. When, however, the frictional drag on the particles becomes equal to their apparent weight, that is the actual weight less the buoyancy force, the particles become rearranged thus offering less resistance to the flow of fluid and the bed starts to expand with a corresponding increase in voidage. This process continues with increase in velocity, with the total frictional force remaining equal to the weight of the particles, until the bed has assumed its loosest stable form of packing. If the velocity is then increased further, the individual particles separate from one another and become freely supported in the fluid. At this stage, the bed is described as *fluidized*.



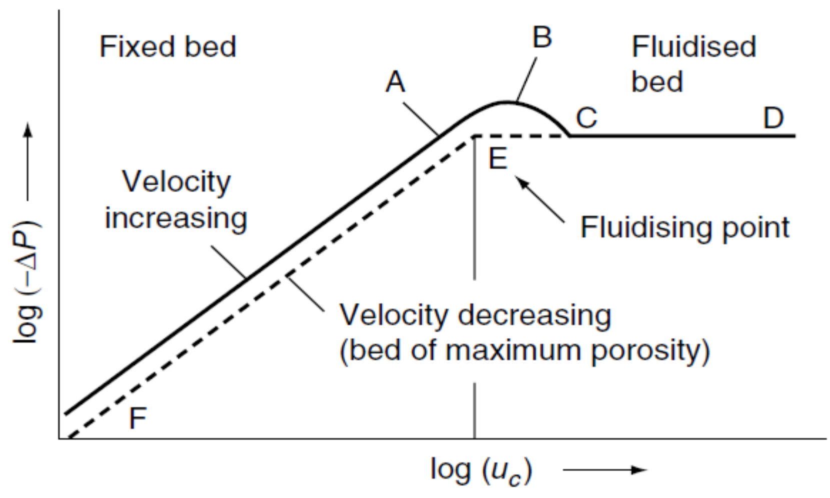
Schematic diagram of fluidization

- At low gas velocities the drag force is to small to lift the bed, which remains fixed. Increasing gas velocity causes solids to move upward and create fluid bed. Depending on the velocity of gas we can distinguish different modes of fluidization (Fig..2) from bubbling fluidization, through turbulent and fast fluidization modes up to pneumatic transport of solids.

FIGURE 1.2: Fluidization type depending on gas velocity



- Another important issue concerning the fluidization process is pressure drop through a fixed bed. The Figure below presents changes in pressure drop with changing gas velocity.



- The velocity at which the pressure is stabilized is called minimum fludization velocity.

In a fluidised bed, the total frictional force on the particles must equal the effective weight of the bed. Thus, in a bed of unit cross-sectional area, depth l, and porosity e, the additional pressure drop across the bed attributable to the layout weight of the particles is given by:

$$-\Delta P = (1 - e)(\rho_s - \rho)lg \tag{6.1}$$

where: g is the acceleration due to gravity and
ρ_s and ρ are the densities of the particles and the fluid respectively.

If flow conditions within the bed are streamline, the relation between fluid velocity u_c, pressure drop $(-\Delta P)$ and voidage e is given, for a fixed bed of spherical particles of diameter d, by the Carman-Kozeny equation (4.12a) which takes the form:

$$u_c = 0.0055\left(\frac{e^3}{(1-e)^2}\right)\left(\frac{-\Delta P d^2}{\mu l}\right) \tag{6.2}$$

For a fluidised bed, the buoyant weight of the particles is counterbalanced by the frictional drag. Substituting for $-\Delta P$ from equation 6.1 into equation 6.2 gives:

$$u_c = 0.0055\left(\frac{e^3}{1-e}\right)\left(\frac{d^2(\rho_s - \rho)g}{\mu}\right) \tag{6.3}$$

6.1.3. Minimum fluidising velocity

As the upward velocity of flow of fluid through a packed bed of uniform spheres is increased, the point of *incipient fluidisation* is reached when the particles are just supported in the fluid. The corresponding value of the *minimum fluidising velocity* (u_{mf}) is then obtained by substituting e_{mf} into equation 6.3 to give:

$$u_{mf} = 0.0055\left(\frac{e_{mf}^3}{1-e_{mf}}\right)\frac{d^2(\rho_s-\rho)g}{\mu} \tag{6.4}$$

Since equation 6.4 is based on the Carman–Kozeny equation, it applies only to conditions of laminar flow, and hence to low values of the Reynolds number for flow in the bed. In practice, this restricts its application to fine particles.

The value of e_{mf} will be a function of the shape, size distribution and surface properties of the particles. Substituting a typical value of 0.4 for e_{mf} in equation 6.4 gives:

$$(u_{mf})_{e_{mf}=0.4} = 0.00059\left(\frac{d^2(\rho_s-\rho)g}{\mu}\right) \tag{6.5}$$

When the flow regime at the point of incipient fluidisation is outside the range over which the Carman-Kozeny equation is applicable, it is necessary to use one of the more general equations for the pressure gradient in the bed, such as the Ergun equation given in equation 4.20 as:

$$\frac{-\Delta P}{l} = 150\left(\frac{(1-e)^2}{e^3}\right)\left(\frac{\mu u_c}{d^2}\right) + 1.75\left(\frac{(1-e)}{e^3}\right)\left(\frac{\rho u_c^2}{d}\right) \tag{6.6}$$

where d is the diameter of the sphere with the same volume:surface area ratio as the particles.
Substituting $e = e_{mf}$ at the incipient fluidisation point and for $-\Delta P$ from equation 6.1, equation 6.6 is then applicable at the minimum fluidisation velocity u_{mf}, and gives:

$$(1-e_{mf})(\rho_s-\rho)g = 150\left(\frac{(1-e_{mf})^2}{e_{mf}^3}\right)\left(\frac{\mu u_{mf}}{d^2}\right) + 1.75\left(\frac{(1-e_{mf})}{e_{mf}^3}\right)\left(\frac{\rho u_{mf}^2}{d}\right) \tag{6.7}$$

Multiplying both sides by $\dfrac{\rho d^3}{\mu^2(1-e_{mf})}$ gives:

$$\frac{\rho(\rho_s-\rho)gd^3}{\mu^2} = 150\left(\frac{1-e_{mf}}{e_{mf}^3}\right)\left(\frac{u_{mf}d\rho}{\mu}\right) + \left(\frac{1.75}{e_{mf}^3}\right)\left(\frac{u_{mf}d\rho}{\mu}\right)^2 \tag{6.8}$$

In equation 6.8:

$$\frac{d^3\rho(\rho_s-\rho)g}{\mu^2} = Ga \tag{6.9}$$

where Ga is the 'Galileo number'.

and:
$$\frac{u_{mf}d\rho}{\mu} = Re'_{mf}. \tag{6.10}$$

where Re_{mf} is the Reynolds number at the minimum fluidising velocity and equation 6.8 then becomes:

$$Ga = 150\left(\frac{1-e_{mf}}{e_{mf}^3}\right)Re'_{mf} + \left(\frac{1.75}{e_{mf}^3}\right)Re'^2_{mf} \tag{6.11}$$

For a typical value of $e_{mf} = 0.4$:

$$Ga = 1406Re'_{mf} + 27.3Re'^2_{mf} \tag{6.12}$$

Thus:

$$Re'^2_{mf} + 51.4Re'_{mf} - 0.0366Ga = 0 \tag{6.13}$$

and:

$$(Re'_{mf})_{e_{mf}=0.4} = 25.7\{\sqrt{(1 + 5.53 \times 10^{-5}Ga)} - 1\} \tag{6.14}$$

and, similarly for $e_{mf} = 0.45$:

$$(Re'_{mf})_{e_{mf}=0.45} = 23.6\{\sqrt{(1 + 9.39 \times 10^{-5}\ Ga)} - 1\} \tag{6.14a}$$

By definition:

$$u_{mf} = \frac{\mu}{d\rho}Re'_{mf} \tag{6.15}$$

It is probable that the Ergun equation, like the Carman-Kozeny equation, also overpredicts pressure drop for fluidised systems, although no experimental evidence is available on the basis of which the values of the coefficients may be amended.

WEN and YU[6] have examined the relationship between voidage at the minimum fluidising velocity, e_{mf}, and particle shape, ϕ_s, which is defined as the ratio of the diameter of the sphere of the same specific as the particle d, as used in the Ergun equation to the diameter of the sphere with the same volume as the particle d_p.

Thus:

$$\phi_s = d/d_p \tag{6.16}$$

where: $d = 6V_p/A_p$ and $d_p = (6V_p/\pi)^{1/3}$.

In practice the particle size d can be determined only by measuring both the volumes V_p and the areas A_p of the particles. Since this operation involves a somewhat tedious experimental technique, it is more convenient to measure the particle volume only and then work in terms of d_p and the shape factor.

... ... point of incipient fluidisation.

Using equation 6.16 to substitute for $\frac{\phi_s}{d_p}$ for d in equation 6.6 gives:

$$(1-e_{mf})(\rho_s-\rho)g = 150\left(\frac{(1-e_{mf})^2}{e_{mf}^3}\right)\left(\frac{\mu u_{mf}}{\phi_s^2 {d_p}^2}\right) + 1.75\left(\frac{1-e_{mf}}{e_{mf}^3}\right)\frac{\rho {u_{mf}}^2}{\phi_s d_p}$$

Thus:

$$\frac{(\rho_s-\rho)\rho g d_p^3}{\mu^2} = 150\left(\frac{1-e_{mf}}{e_{mf}^3}\right)\frac{1}{\phi_s^2}\left(\frac{\rho d_p u_{mf}}{\mu}\right) + 1.75\left(\frac{1}{e_{mf}^3\phi_s}\right)\left(\frac{\rho^2 d_p^2 u_{mf}^2}{\mu^2}\right)$$

Substituting from equations 6.17 and 6.18:

$$Ga_p = (150 \times 11)Re'_{mfp} + (1.75 \times 14)Re'^2_{mfp}$$

where Ga_p and Re_{mfp} are the Galileo number and the particle Reynolds number at the point of incipient fluidisation, in both cases with the linear dimension of the particles expressed as d_p.

Thus: $$Re'^2_{mfp} + 67.3Re'_{mfp} - 0.0408Ga_p = 0$$

giving: $$Re'_{mfp} = 33.65[\sqrt{(1+6.18\times10^{-5}Ga_p)} - 1] \tag{6.19}$$

where: $$u_{mf} = \left(\frac{\mu}{d_p\rho}\right)Re'_{mfp} \tag{6.20}$$

For small particles $Re_{,p,mf} < 20$

$$\Rightarrow U_{mf} = \frac{{d_p}^2(\rho_p-\rho_f)g}{150\,\mu_f}\left(\frac{\epsilon_{mf}^3\ \phi_s^2}{1-\epsilon_{mf}}\right)$$

For large particles $(Re_{,pmf} > 1000)$

$$\Rightarrow U_{mf}^2 = \frac{d_p(\rho_p-\rho_f)g}{1.75\,\rho_f}\ \epsilon_{mf}^3\ \phi_s$$

To avoid or reduce carryover of particles form the fluidized bed, keep the gas velocity between U_{mf} and U_t. Recall

Terminal velocity, $u_t = \frac{gd_p^2(\rho_p-\rho_f)}{18\mu_f}$ for low Reynolds number and,

$U_t = 1.75\frac{\sqrt{gd_p(\rho_p-\rho_f)}}{\rho_p}$ for high Reynolds number

With the expressions for U_{mf} and U_t known for small (viscous–flow) and large (inertial flow) particles or Reynolds number, one can take the ratio of U_t and U_{mf}:

For small $Re_{,P}$: $$\frac{U_t}{U_{mf}} = \frac{150\,(1-\epsilon_{mf})}{18\,\epsilon_{mf}^2\,\phi_s^2} = 8.33\frac{(1-\epsilon_{mf})}{\phi_s^2\,\epsilon_{mf}^2}$$

For spherical particles, $\phi_s = 1$ and assuming $\varepsilon_{mf} = 0.45, U_t = 50\,U_{mf}$

Therefore, a bed that fluidizes at 1cm/s could preferably be operated with velocities < 50 cm/s, with few particles carried out or entrained with the exit gas.

For large $Re_{,P}$: $$\frac{U_t}{U_{mf}} = \frac{2.32}{\epsilon_{mf}^{1/2}}$$

Or, $u_t = 7.7\,u_{mf}$ for $\varepsilon_{mf} = 0.45$,

Therefore, operating safety margin in a bed of coarse particles is smaller and there is a disadvantage for the use of coarse particles in a fluidized bed.

However, make a note that the operating particle size is also decided by the other factors such as grinding cost, pressure-drop, heat and mass-transfer aspects.

Example 6.1

A bed consists of uniform spherical particles of diameter 3 mm and density 4200 kg/m^3. What will be the minimum fluidising velocity in a liquid of viscosity 3 mNs/m^2 and density 1100 kg/m^3?

Solution

By definition:

$$\text{Galileo number, } Ga = d^3\rho(\rho_s - \rho)g/\mu^2$$
$$= ((3 \times 10^{-3})^3 \times 1100 \times (4200 - 1100) \times 9.81)/(3 \times 10^{-3})^2$$
$$= 1.003 \times 10^5$$

Assuming a value of 0.4 for e_{mf}, equation 6.14 gives:

$$Re'_{mf} = 25.7\{\sqrt{(1 + (5.53 \times 10^{-5})(1.003 \times 10^5))} - 1\} = 40$$

and: $$u_{mf} = (40 \times 3 \times 10^{-3})/(3 \times 10^{-3} \times 1100) = 0.0364 \text{ m/s or } \underline{\underline{36.4 \text{ mm/s}}}$$

Example 6.2

Oil, of density 900 kg/m^3 and viscosity 3 mNs/m^2, is passed vertically upwards through a bed of catalyst consisting of approximately spherical particles of diameter 0.1 mm and density 2600 kg/m^3. At approximately what mass rate of flow per unit area of bed will (a) fluidisation, and (b) transport of particles occur?

Solution

(a) Equations 4.9 and 6.1 may be used to determine the fluidising velocity, u_{mf}.

$$u = (1/K'')(e^3/(S^2(1-e)^2)(1/\mu)(-\Delta P/l) \qquad \text{(equation 4.9)}$$

$$-\Delta P = (1-e)(\rho_s - \rho)lg \qquad \text{(equation 6.1)}$$

where S = surface area/volume, which, for a sphere, $= \pi d^2/(\pi d^3/6) = 6/d$.

Substituting $K'' = 5$, $S = 6/d$ and $-\Delta P/l$ from equation 6.1 into equation 4.9 gives:

$$u_{mf} = 0.0055(e^3/(1-e))(d^2(\rho_s - \rho)g)/\mu$$

Hence : $$G'_{mf} = \rho u = (0.0055e^3/(1-e))(d^2(\rho_s - \rho)g)/\mu$$

In this problem, $\rho_s = 2600$ kg/m^3, $\rho = 900$ kg/m^3, $\mu = 3.0 \times 10^{-3}$ Ns/m^2 and $d = 0.1$ mm $= 1.0 \times 10^{-4}$ m.

As no value of the voidage is available, e will be estimated by considering eight closely packed spheres of diameter d in a cube of side $2d$. Thus:

$$\text{volume of spheres} = 8(\pi/6)d^3$$

$$\text{volume of the enclosure} = (2d)^3 = 8d^3$$

and hence: voidage, $e = [8d^3 - 8(\pi/6)d^3]/8d^3 = 0.478$, say, 0.48.

Thus : $$G'_{mf} = 0.0055(0.48)^3(10^{-4})^2((900 \times 1700) \times 9.81)/((1-0.48) \times 3 \times 10^{-3}$$

$$= \underline{\underline{0.059 \text{ kg/m}^2\text{s}}}$$

(b) Transport of the particles will occur when the fluid velocity is equal to the terminal falling velocity of the particle.

Using Stokes' law : $$u_0 = d^2 g(\rho_s - \rho)/18\mu \qquad \text{(equation 3.24)}$$

$$= ((10^{-4})^2 \times 9.81 \times 1700)/(18 \times 3 \times 10^{-3})$$

$$= 0.0031 \text{ m/s}$$

The Reynolds number $= ((10^{-4} \times 0.0031 \times 900)/(3 \times 10^{-3}) = 0.093$ and hence Stokes' law applies.

The required mass flow $= (0.0031 \times 900) = 2.78$ kg/m^{2}s

An alternative approach is to make use of Figure 3.6 and equation 3.35,

$$(R/\rho u^2)Re^2 = 2d^3\rho g(\rho_s - \rho)/3\mu^2$$

$$= (2 \times (10^{-4})^3 \times (900 \times 9.81) \times 1700)/(3(3 \times 10^{-3})^2) = 1.11$$

From Figure 3.6, $Re = 0.09$

Hence: $$u_0 = Re(\mu/\rho d) = (0.09 \times 3 \times 10^{-3})/(900 \times 10^{-4}) = 0.003 \text{ m/s}$$

and: $$G' = (0.003 \times 900) = \underline{\underline{2.7 \text{ kg/m}^2\text{s}}}$$

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