



CONTROL SYSTEMS



1.1 Introduction:

A Control System is an arrangement of different physical elements connected in such a manner so as to regulate, direct or command itself or some other system. The general form is shown in Fig.(1-1).

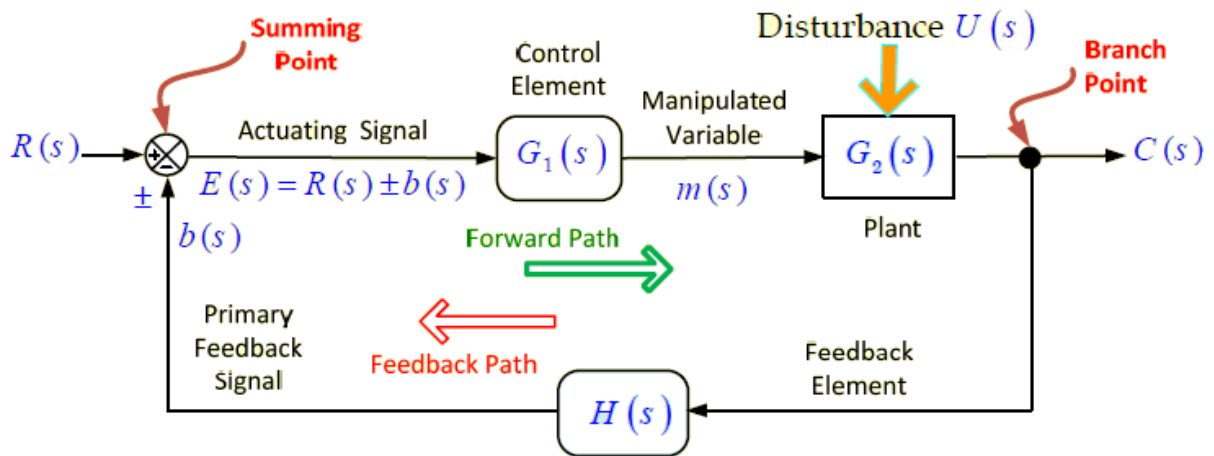


Fig.(1-1) Closed Loop Control System.

Plant or Process $G_2(s)$ is a physical object to be controlled.

Controller $G_1(s)$ is the element of the system which control the plant.

Feedback element $H(s)$ is the component required to establish the functional relationship between the primary feedback signal $B(s)$ and the controlled output $C(s)$.

Reference Input $R(s)$ is an external signal applied to a feedback control system in order to command a specified action of the plant.

Open loop control system: output of the system is not feedback to the system.

Closed Loop Control System: a system which compares output to some reference input and keeps output as possible to this reference.

Forward Path is the transmission path from the actuating signal $E(s)$ to the output $C(s)$.

Feedback Path is the transmission path from the output $C(s)$ to the feedback signal $B(s)$.

Disturbance $U(s)$ is an undesired input signal which affects the value of the controlled output $C(s)$.



$G(s) H(s)$ = Open Loop Transfer Function
 $1+G(s) H(s)$ = Characteristic Equation
 $C(s)/R(s)$ = Closed Loop Transfer Function
 $E(s)/R(s)$ = Error Transfer Function

1.2 Classification of Control Systems:

System models are classified according to the types of the equations used to describe them. The family tree shown in Fig.(1-2) illustrates the major system classifications. Combination of these classes can also occur.

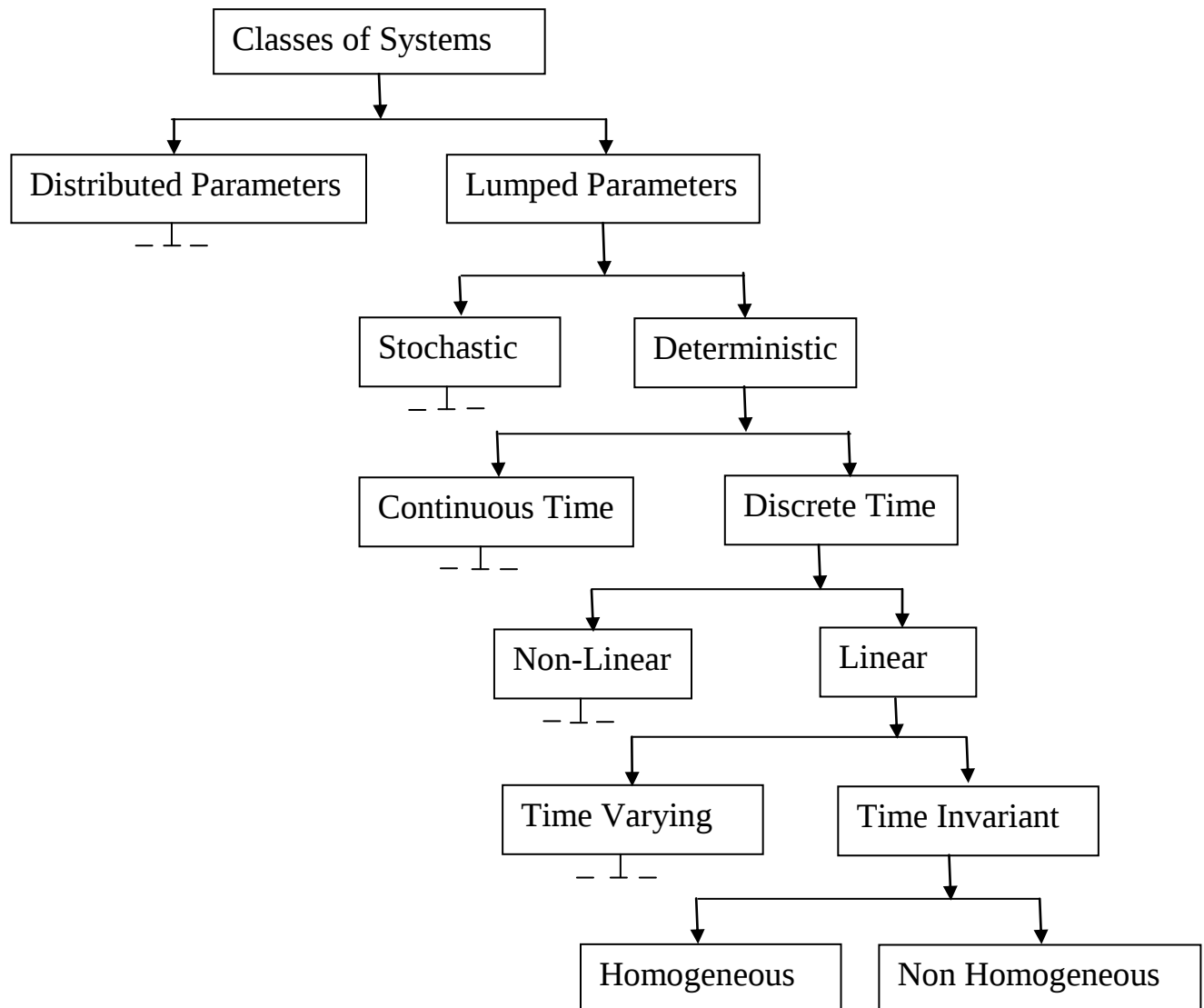


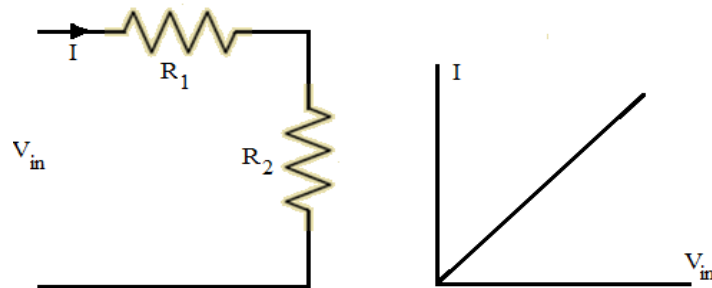
Fig.(1-2) Tree of System Classifications.

Distributed parameter systems require Partial Differential Equations (PDE) for their description. Lumped parameter systems are those for which all energy storage or dissipation can be lumped into a finite number of discrete spatial locations. They are described by Ordinary Differential Equations (ODE). Discrete Component electric circuit falls into this category.

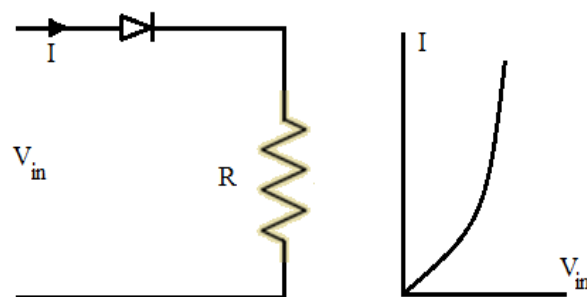
Systems containing parameters or signals which can only be described in probabilistic fashion are called stochastic or Random systems. If all elemental equations are defined for all time, then the system is a continuous time system. If, as in sampling or digital systems, same elemental equations are defined or used only at discrete points in time, a discrete time system is the result. Continuous time systems are described by ODE, while the discrete time system by differential equations.

If all elemental equations are linear, so is the system. If one or more elements are nonlinear then the overall system is nonlinear.

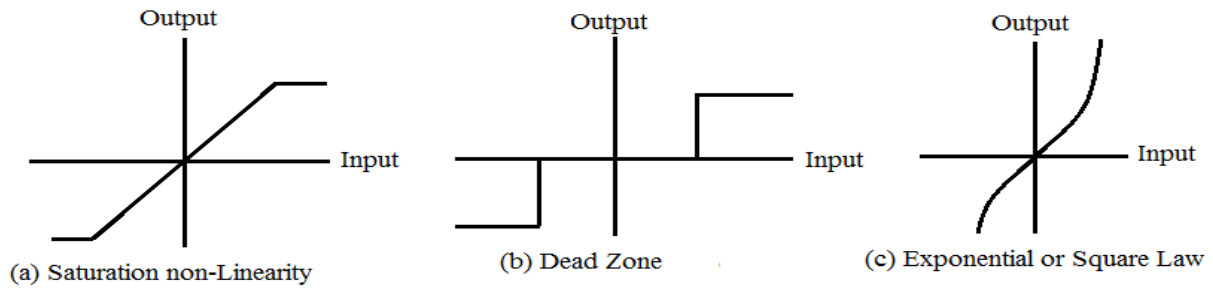
Example of Linear System: Linear relationship between the applied voltage V_{in} and input current I .



Example of non-Linear system:



Example of non-Linear elements:



Example of non-Linear functions:

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$$

1.3 Open Loop System:

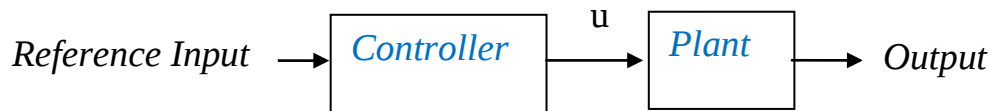


Fig.(1-3) Open Loop System.

Advantages: simple construction, easy maintenance, simple design, very much convenient when output is difficult to measure.

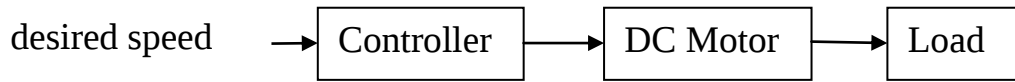
Disadvantages: inaccurate results if there are variations in the system parameters, recalibration of the controller is necessary, cannot sense the internal disturbances in the system.

The closed loop system shown in Fig.(1-1) has advantages:

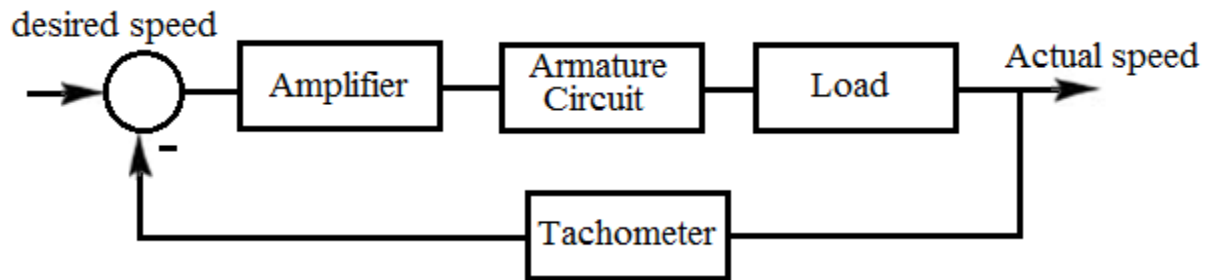
- High accuracy because the controller modifies and manipulates the actuating signal such that the system error will be zero.
- The closed loop system senses the environmental changes as well as internal disturbances.
- In such a system, there is reduced effect of nonlinearities.

Example: Speed Control of DC Motor

(a) Open Loop System:



(b) Closed loop control:



1.4 Laplace Transform:

Let $f(t)$ be a real function with $t > 0$ then $F(s) = \int_0^{\infty} f(t)e^{-st} dt$

Where $F(s)$ is called Laplace Transform of $f(t)$ and $s = \sigma + j\omega$

Example:

$$f(t) = 1$$

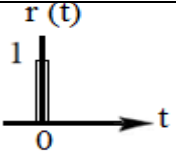
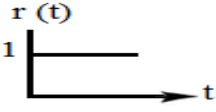
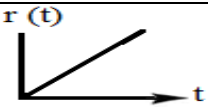
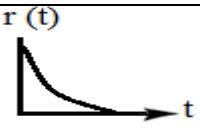
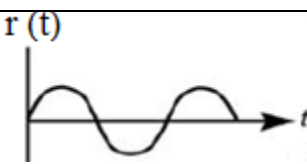
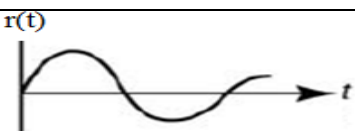
$$F(s) = \int_0^{\infty} f(t)e^{-st} dt = \int_0^{\infty} e^{-st} dt = \left[-\frac{1}{s} e^{-st} \right]_0^{\infty} = 0 - \frac{-1}{s} = \frac{1}{s}$$

Example:

$$f(t) = e^{-at}$$

$$\begin{aligned} F(s) &= \int_0^{\infty} f(t)e^{-st} dt = \int_0^{\infty} e^{-at} e^{-st} dt = \int_0^{\infty} e^{-t(s+a)} dt \\ &= \left[-\frac{1}{(s+a)} e^{-t(s+a)} \right]_0^{\infty} = 0 - \frac{-1}{(s+a)} = \frac{1}{s+a} \end{aligned}$$

Table (1-1) Laplace Transform.

Time Function	Plot	Laplace Transform
$r(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$		$F(s) = 1$
Unit step $r(t) = 1$		$F(s) = \frac{1}{s}$
Unit Ramp $r(t) = t$		$F(s) = \frac{1}{s^2}$
$r(t) = t^n$ where $n = 1, 2, \dots$		$F(s) = \frac{n!}{s^{n+1}}$
$r(t) = e^{-at}$		$F(s) = \frac{1}{s + a}$
$r(t) = t^n e^{-at}$ where $n = 1, 2, \dots$		$F(s) = \frac{n!}{(s + a)^{n+1}}$
$r(t) = \sin(\omega t)$		$F(s) = \frac{\omega}{s^2 + \omega^2}$
$r(t) = \cos(\omega t)$		$F(s) = \frac{s}{s^2 + \omega^2}$
$r(t) = e^{-at} \sin(\omega t)$		$F(s) = \frac{\omega}{(s + a)^2 + \omega^2}$
$r(t) = e^{-at} \cos(\omega t)$		$F(s) = \frac{s + a}{(s + a)^2 + \omega^2}$

Table(1-2) Laplace Properties.(LT = Laplace Transform).

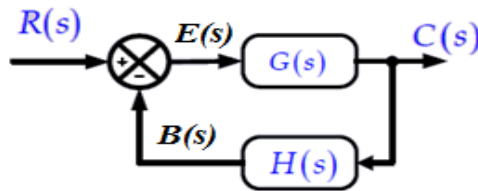
1. Magnification	$LT\{Af(t)\} = AF(s)$
2. Addition	$LT\{f_1(t) + f_2(t)\} = F_1(s) + F_2(s)$
3. First Order Derivative	$LT\{\dot{f}(t)\} = sF(s) - f(0)$
4. Second Order Derivative	$LT\{\ddot{f}(t)\} = s^2F(s) - sf(0) - \dot{f}(0)$
5. Integral	$LT\left\{\int_0^t f(t)dt\right\} = \frac{F(s)}{s}$
6. Time Multiplication	$LT\{tf(t)\} = \frac{-dF(s)}{ds}$
7. Exponential Decay	$LT\{e^{-at}f(t)\} = F(s+a)$
8. Initial Value Theorem	$f(0) = \lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} sF(s)$
9. Final Value Theorem	$f(\infty) = \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$

1.5 Transfer Function:

It is defined as the ratio of Laplace transform of system output to the Laplace transform of the system input.

$$G(s) = \frac{C(s)}{R(s)}$$

Consider a simple closed loop system using negative feedback as:



$$E = R - B = R - CH = R - EGH$$

$$E(1 + GH) = R$$

$$E = \frac{R}{1 + GH}$$

also

$$C = E G = \frac{R G}{1 + GH} \quad \text{or} \quad \frac{C(s)}{R(s)} = \frac{G}{1 + GH}$$

But with positive feedback $\frac{C(s)}{R(s)} = \frac{G}{1 - GH}$